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RESEARCH ARTICLE

Enhancing Students' Understanding of Linear Equation With One Variable Through Teaching

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ABSTRACT

The objective of this study is to enhance students' understanding of solving linear equation with one variable through teaching using balancing model. To achieve this research objective, design-based research approach was chosen. The target population of the study was grade five students and their mathematics teacher. From this population, the participants of the study were grade five section A students and their respective mathematics teacher who were selected using simple random sampling and comprehensive sampling techniques, respectively. The data were collected through observation, teacher made tests and interview. The test results were analyzed using paired samples t-test and percentage, and the interview and classroom observation data were analyzed through thematic description. Findings showed that most students performed better in the post-test as compared to the pre-test, and most students came to use more flexible strategies to solve linear equation after a series of learning attempts made using balancing model. Moreover, the students and their mathematics teacher showed positive attitude towards learning linear equation through balancing model. On the basis of these findings, the researchers have forwarded relevant recommendations in the paper.

1. INTRODUCTION.

Mathematics is a culturally shared study of patterns and language for everyday life, a central part of human communication and a means of articulating patterns, relationships, rationality and aesthetic (Jaworski, 2006). That is, mathematics concepts connect one to his or her universe in many ways by incorporating language, culture and practices of his/her daily living. That is why the goal of mathematics education is directed towards producing students who are skilled in resolving problems, fostering attitudes, interests and a high motivation towards mathematics. In this regard, Anthony and Walshaw (2014) once asserted that the main purpose of teaching and learning mathematics subject should be to develop the ability of students and connect any mathematical problems to the students' real-world situations. On the other hand, mathematical ability involves effective thinking with conceptual understanding, thus students need to be taught to think logically along with practicing numerical problems (Mahmood, et al., 2012).

The concept of algebra is the major component of the mathematics curriculum in all countries around the world (Bal, 2016). Therefore, understanding algebra in school mathematics is one of the most important goals of teaching mathematics. Algebra concepts include the unknowns, variables, expressions, equations and mathematical operations. Moreover, algebra deals with expressions with symbols and the extended numbers beyond whole numbers in order to solve equations, to analyze functional relations- and to determine the structure of the representational system that consists of expressions and relations. Bal (2016) further stated that primary school algebra that involves arithmetical skills and numerical patterns has great importance to enhance the students' algebraic understanding.

Students learn algebra concepts starting from arithmetical operations and move towards solving linear equation. Linear equation is one of the most important topics in algebra and it also played a central role in the development of other mathematics concepts. However, students from many countries displayed relatively negative attitude towards mathematics particularly algebra concepts (Hescovics & Linchevski, 1994). As the researchers' lived experience confirmed, this also holds true in the Ethiopian context. The study conducted by the Ministry of Education (2003), for example, disclosed that mathematics teaching and learning is not promising in the Ethiopian schools. Some of the mathematics instruction problems include dominantly using the traditional teaching approaches, lack of reference materials, lack of professional development programs organic to the school system, and absence of encouraging students' prior experiences (Ministry of Education, 2003).

However, researchers attest that alternative teaching styles and strategies, and bridging the transition from concrete to abstract mathematics could promote positive attitude among students.

Therefore, selection of relevant teaching methods by taking into account the nature of the subject matter, the objectives of the lesson and the nature of the learners is one of the primary principles in mathematics education. According to Bal (2016), applying different teaching strategies to mathematics lessons help the students to value the lessons more, learned with fun and be able to solve various problems with different methods. In this regard, many studies pointed out that a balanced model teaching method is an effective means to develop students' understanding of the relational view of the equal sign and their ability to operate on equations. Warren and Cooper (2005), for example, noted that balance model is an innovative tool that makes algebra more accessible for students. By working in cooperative groups using the balance model, the students can improve communication skills as they explore algebraic concepts together. In algebra, a balance scale can be a powerful tool to understand the idea of an equation. In this regard, Atteh et al., (2017) disclosed some advantages of the balance model in teaching linear equation including, promoting the view of an equation as an object, facilitating understanding of eliminations in algebraic operation, and increasing representational fluency.

Therefore, this study is aimed at building up a local instructional theory in teaching algebra, particularly on the topic of linear equation with one variable. To address this aim, the researchers' have designed learning materials and learning activities using balancing model to promote the students understanding and reasoning power about linear equation. The design makes most of the algebraic notions within balancing activity to facilitate students' learning of linear equation with one variable. So, the findings of this study is hoped to provide a practical solution for teachers to reform their algebra teaching in the Ethiopian primary schools.

Statement of the Problem

Research findings and experience in teaching mathematics showed multiple difficulties that students encounter in learning algebra (Kilpatrick & Izsak, 2008). The students' difficulties are caused by several factors including the way algebra is taught, mathematics teachers teaching style as well as most students failure of experiencing practical learning (Eichhorn, *et al.*, 2018). As a result, learning primary school algebra particularly linear equation is the major mathematical challenge in the operational stage to most students (Cai & Moyer, 2008). There are many reasons why solving linear equations is a challenging skill for students to understand. According to Kieran (2003), students learning difficulties are centered on the meaning of the equal sign, lack of symbolic understanding, the change from arithmetic to algebraic conventions and use of a structure. So students without a relational view of the equal sign cannot adequately interpret the linear equation. As a result, many students do not have the capacity to construct a good intuitive basis for the concept of an equation and they fail to construct meaning for the new symbolism and perform meaningless operations on symbols they do not understand.

Mathematics teachers in Ethiopia failed to teach using active learning methods for exploiting the advantages of promoting learners' learning styles and strategies (MoE, 2003). In addition, the researchers' practical experiences as mathematics teachers confirmed that students' different learning styles and strategies were not considered in the teaching of mathematics. Moreover, the Ethiopian mathematics textbooks failed to include innovative approaches (e.g., using balancing model). Therefore, most Ethiopian students consider mathematics as one of the most difficult subject. The researchers' preliminary observation of Ewket Fana Primary School also confirmed that the existences of theoretical and practical gaps mentioned above. The researchers' discussion with grade five mathematics teacher further showed that he failed to use the balancing model teaching method during his teaching-learning process and also his students did not experience the balancing model teaching method for solving any linear equation. The main reasons for not applying balancing model teaching approach in this school were lack of pedagogical knowledge of the teacher, textbook writers' failure to include the approach while designing the textbooks and lack of resources.

Therefore, to develop a relational and conceptual understanding of the equal sign and to support students' mathematical thinking and algebraic reasoning in elementary schools, teachers should apply innovative teaching methods. In this regard, balancing model teaching method is considered as an appropriate teaching method in solving linear equation problems in the current mathematics literature. That is, different studies (e.g., Vlassis, 2002; Atteh et al., 2017) disclosed the effectiveness of balancing model method of teaching in learning about the concepts of solving linear equation. According to Vlassis (2002), learning linear equation by using the balance model has motivated the students to relate mathematical concepts with observation and exploration data and it can be used for bridging the gaps in understanding the abstract concepts in algebra. Similarly, Atteh et al., (2017) unfold that the balance model also promotes the active participation and intellectual involvement of learners in solving a linear equation. Based on these background evidences, the researchers felt that balancing model teaching method plays an important role in developing active learning and improve the conceptual understanding of linear equation among students.

Theoretical Framework of the Study

This study drew its theoretical framework from constructivism, which has two belief systems: individual constructivism and social constructivism. The theory of individual constructivism viewed learning as a process in which the learner actively constructs or builds new ideas or concepts based on current and past knowledge and experience (Von Glasersfeld, 1990). However, social constructivism is considered as a complement to the traditional focus on individual learning to address the collaborative and social dimension of learning (Ernest, 1994). Social constructivists posit that knowledge is constructed when individuals engage socially in talk and activity about shared problems or tasks. From these conceptualizations, one can see that this theory suggest that learners mind is not like an empty vessel that has to be filled-in with knowledge but that learners are active beings who are capable of servicing of new knowledge from known and related experience and also through social interaction with other learners in a group. The implication behind this theory is creating active learning through collaborative and cooperative group discussion environments.

Conceptual framework

The study was guided by a conceptual framework depicting the representation of dependent and independent variables and the relationships between them as shown in figure 1.

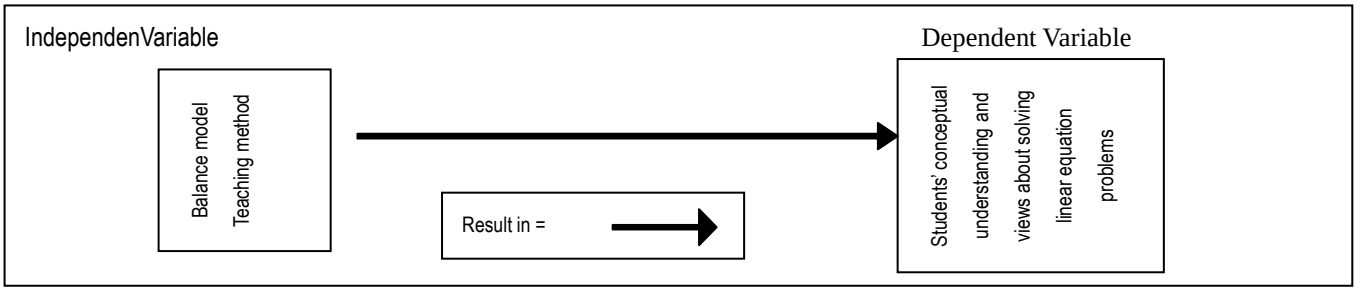


Figure 1. Conceptual Framework of the Study

The central thesis of the study was to check how far the selected teaching method (i.e., balance model teaching method or innovative method) has contributed to grade five students' conceptual understanding of solving linear equation and changing their views about the nature of learning of linear equation.

Objective of the Study

The main objectives of this study are:

- to enhance students' conceptual understanding of solving linear equation with one variable through balancing model method of teaching, and
- to sustain the balancing model method of teaching approach in teaching the concept of solving linear equation with one variable in Ewket Fana Primary School.

Research Questions of the Study

The basic research questions in this study include the following:

- Do students' show better conceptual understanding of solving linear equation when they are taught using the balancing model method?
- How can we sustain balancing model teaching method in teaching the concept of solving linear equation with one variable in Ewket Fana Primary School?

Significance of the Study

This study is hoped to provide the following significances:

- be used as a reference point for curriculum experts to train the new approach for mathematics teachers and even re-design all mathematics textbooks,
- be used as a guideline for mathematics teachers to design effective instructional activities for teaching the concept of solving linear equation and engaging their students in a sequence of instructional activities in order to support their conceptual understanding about linear equation, and
- will serve as baseline data for future researchers who want to do large scale investigation in the contribution of teaching using balancing model for students' conceptual understanding of linear equation.

The Scope of the Study

The scope of the study was delimited to one government primary school (Ewket Fana) of Bahir Dar Town. Conceptually, the study was delimited to enhancing students' understanding of solving linear equation problems and checking how far this will be sustained.

2. METHODS

Research Design

This study used quasi-experimental single-group design based research. This single-group design allows the same group to be compared over time by considering the trend of the data before and after the treatment. Moreover, this study employed both quantitative and qualitative approaches.

Design of the Intervention Plan

To answer the research questions, the researchers planned a design of learning sequences equipped with teaching and learning materials and design the lessons into twelve days lesson. Therefore, the twelve days intervention plans have been designed as in the following table 1.

Table 1. Intervention plan for each topic on solving linear equation

Days	Topic	Learning outcome: at the end of this lesson, students will be able to:
One	Pre-test	Take the pre-test
Two and three	Meaning of equation and equation formulation	Define the meaning of equal relation. Establish an equality statement.
Four and five	Addition of equation in the form of; x + b =c.	Find the value of the unknown number. Find the general formula of the equation; x + b = c.
Six and seven	Subtraction of equation in the form of; x - b = c	Find the value of the unknown number. Find the general formula of the equation; x – b = c.
Eight and nine	Multiplication of equation in the form of ax = c	Find the value of the unknown number. Find the general formula of the equation; ax = c.
Ten and eleven	Multi-step equation	Find the value of the unknown number. Find the general formula of the equation; ax + b=c.
Twelve	Post-test	Take the post-test.

According to Table1, the researchers conducted the teaching experiment for ten days by preparing an intervention plan. Before the implementation of the teaching experiment, the researchers administered the pre-test for the participants. After administering the pre-test, the researchers implemented the intervention by preparing the lesson plan using balancing model teaching method. While walking around and instructing students during the lesson, one of the researchers asked students to explain their work related to balancing model teaching method. In design based research, therefore, the theory and the practice are intertwined to develop theories about the learning process and the activities or tools that can support students' learning.

Procedure of the Research Design

In order to achieve the objective of the study, the researchers designed instructional activities based on the constructivist theory of learning, tried and revised each lesson. This approach has three main phases including preparing for the experiment, teaching experiment in the classroom, and conducting a retrospective analysis.

Phase 1: Preparation and design phase

The purpose of the preparation phase was to analyze the practical problems. In this study, the researchers reviewed some studies related to the problem that usually happened in algebra classes, particularly in the Ethiopian classrooms. This was followed by analyzing the Ethiopian curriculum and its components, such as the textbooks, a list of topics and teachers' guide released by the Ministry of Education of Ethiopia. In addition, the researchers collected preliminary data about the characteristic and prior knowledge of the participant students related to linear equation concepts. Combining all these information, the researchers came-up with critical causes of the students' difficulties in learning linear equation. Then after, the researchers reviewed other studies that focus on how to teach linear equation, to figure out what the students will have during their linear equation studies. Finally, we developed learning materials. This included lesson plans, teacher guides, and students worksheets. The next step was to test these conjectures in the teaching experiment phase.

Phase 2: Teaching experiment phase

The main objective of the second phase was to test and improve the conjecture that is already made in the preparation phase and to enhance an understanding of how it works. In this phase, the researchers implemented the sequence of instructional activities designed in the preparation phase. Due to the cyclic nature of the design based research, the teaching experiment phase had one macrocycle containing a full series of learning. The macrocycle consists of micro cycles. During this teaching experiment, the researchers collected data through observation, students' worksheet and students' reflection. After the implementation of one microcycle, the learning process was analyzed to adjust the continuity of the learning sequence. This learning was also analyzed during the micro cycle and after the macrocycle. At the closing session of the teaching experiment phase, the researchers gave post-test to the students.

Phase 3: Retrospective analysis phase

In the design based research, the retrospective analysis phase aims to review, observe and interpret what was happening in the classroom during the teaching experiment phase. This phase was done by comparing the hypothesized students' learning with the actual conditions during the teaching experiment. In this study, the researchers used qualitative analysis of the observation and interview data, and get an insight about the students' level of understanding the concept of linear equation, and the improvements students' and teacher's point of view about the innovative approach as a result of the learning process. In addition, the teacher made tests were examined the students' conceptual change after the intervention. The researchers compared the result of the pre-test and post-test results to explain how the innovative teaching approach supports students' conceptual understanding of solving linear equation.

Population of the Study

This study was conducted at X Primary School, grade five students (with a total number of 223, M = 109 & F = 114) and their mathematics teacher.

Sample Size and Sampling Technique

One section (with a total of 48) out of five sections of grade five students and their respective mathematics teacher were selected as samples of this study using random and comprehensive sampling techniques, respectively.

Data Gathering Instruments and Procedures

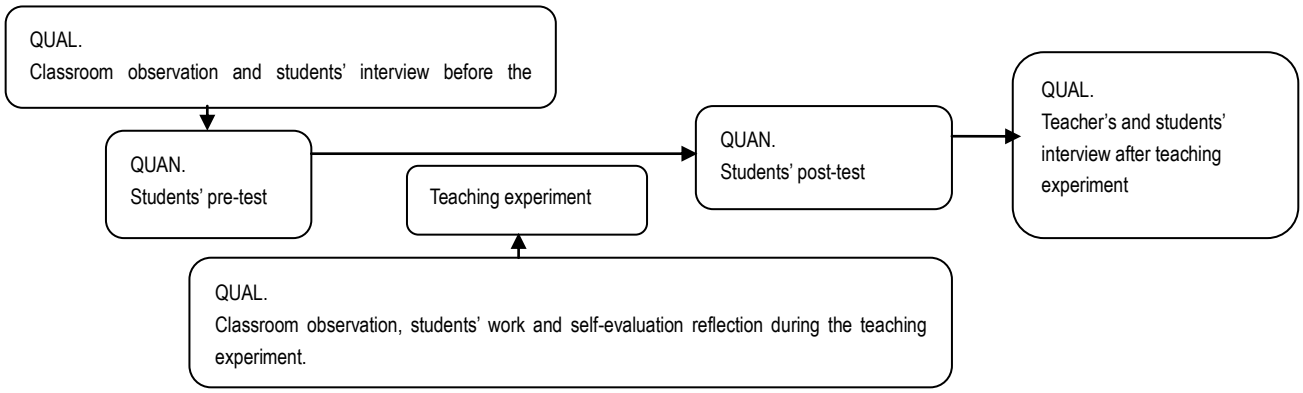


Figure 2. Data Gathering Procedures Of The Study

Preparation phase

Collecting data in this phase mainly aims to provide sufficient information about the students' prior background. This is important to reveal the starting points of the students and the applicability of the design in the classroom, and to make an adjustment with the initial step in our hypothetical learning theory.

Classroom observation before the teaching experiment

In the preparation phase, the researchers collected data through observation. This data gathering tool helped to get a general view of the motivation and participation of the students in the previous lesson and to get a general overview about whether or not the students mastered the previous lesson. Generally, the purpose of this observation was to find out the students status about the previous topic.

Pre-test

After observing the teaching-learning process, the researchers administrated pre-test before conducting the teaching experiment. The pre-test questions mainly aimed at revealing two things, i.e., to know the students' understanding of the prerequisite knowledge and to find out the students' understanding of the topic of the lesson.

Students' pre-intervention interview

After conducting the classroom observation and administered the pre-test, the researchers interviewed five students. The purpose of this interview was to collect data related to students' attitude towards mathematics subject in general and linear equation in particular.

Data collection during the teaching experiment phase

The data gathered during this phase was expected to describe how the students learned during five series of lessons and how far the design helped students to learn. In this phase, the researchers collected data using classroom observation and students reflection. In addition, the researchers collected data at the end of the teaching experiment using post-test.

Classroom observation

In the teaching experiment phase, the researchers collected data using observation to check the students' motivation and attitude towards learning linear equation using the current approach. The observation was supported with a checklist. Moreover, this data gathering tool was intended to describe preconceived notions related to the benefits and challenges attributed to each treatment, concerns related to using balancing model teaching and to identify the strength and limitation of this teaching method.

Post-test

After conducting the teaching experiment, the researchers administrated the post-test. The post-test consists of linear equation problems, as it is used to examine the students' conceptual change after the teaching experiment in the area of solving linear equation with one variable.

Post-intervention interview

At the end of the lessons, the researchers conducted an interview with the teacher and students. The students' interview questions were designed to match with the initial interview questions and the teacher interview was related to the importance of the balancing model teaching method. Generally, the purpose of this interview was to find out the perceived benefits attributed to each treatment, and teacher and students view about the balancing model method of teaching approach.

Data Analysis

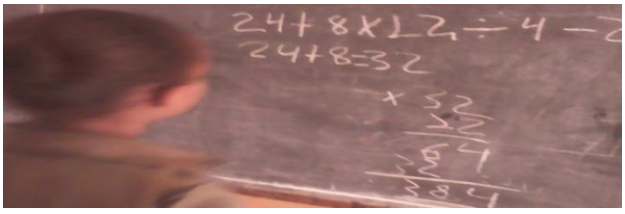
In this study, the researchers analyzed the quantitative data by comparing the pre- and post-test results of each student using paired samples t-test analysis to determine whether or not there exists statistically significant difference in performance of each student before and after the teaching experiment. In addition, the researchers analyzed the observation data using data-driven thematic description to check the sustainability of balancing model teaching method in teaching the concept of solving linear equation with one variable.

3. RESULTS AND DISCUSSION

Students' Understanding of Algebraic Expression

Classroom observation before the teaching experiment

Before the experiment, the researchers observed the teaching-learning process for two consecutive lessons. When the researchers joined the classroom for the purpose of pre-intervention observation, the students were learning about 'simplification of an algebraic expression'. In order to give an overview of what was happening during the pre-intervention lessons, one question each from the two lessons were brought as samples to this presentation. The questions clearly show that the conversation between the teacher and students during the two lessons. In the first day, the students were learning about how to simplify the expression: 24 + 8 x 12 ÷ 4 – 2. One of the students answer is given as follows.



24 + 8 x 12 ÷ 4 - 2=
32 x 12 ÷ 4 -2……..add 24 and 8
384 ÷ 4 – 2……multiplied 32 by 12
96 – 2 ……divided 384 by 4
94……subtracted from 94 to 2.

Figure 3. An example of the student's misconception in simplifying an algebraic expression

As it is shown in figure 3, the student's answer was 94 though the correct answer was 46. This indicates that the students did not understand the order of operation rule. Instead, the students' level of understanding was to perform multiple-operations in any order which appears possible for them. In the second lesson, the students were learning about simplifying the following algebraic expression: 'simplify the expression x2 – 5x; where the value of x is equal to 4'. The students solved this question by using the following procedure:

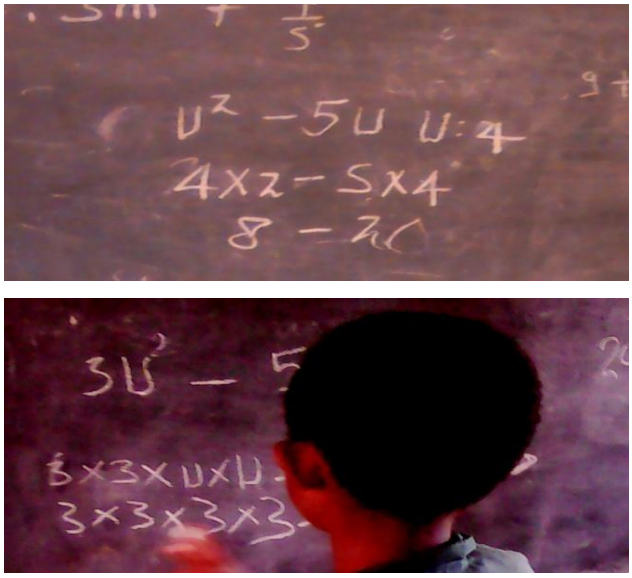


Figure 4. An example of the students' misconception of simplifying an algebraic expression

Generally, the preparation phase observation data showed that the teacher used the traditional teaching method, i.e., talk and chalk strategies and failed to use different teaching aids or alternative strategies. During this time, most of the students were passive listeners, i.e., they did not participate actively in the teaching-learning process. During this phase, instructional message flows only from the teacher to the students. Moreover, the students did not justify their work as the teacher failed to use varieties of teaching methods and was unable to encourage the students by giving different activities that could help them to develop conceptual understanding of the lesson.

Pre-Test Results

Problem 1: 'Write the following word problem within a mathematical algebraic expression, i.e., the difference between twice of one unknown number and four.'

The way the textbook is organized and the preparation phase observation data indicated that the students had learned about the concept of algebraic expressions and the formulation of an algebraic expression statement in the previous lesson. In line with what they had learned, the result of the pre-test indicated that only 5 (10.5 %) students understood the concept of the problem and correctly changed the given word problem into a mathematical formula. Most of the students (43 (89.5 %)) failed to change this word problem into a mathematical formula. Only 5 students did the task by using mathematical symbols but most of the students were deficient in abstract reasoning, language acquisition and mathematical structure during the learning of algebraic expressions.

Problem 2: 'find the value of the given expression; 3 + y x 2, given the value of y = 4?'

The aim of this problem was to check the students' understanding of the order of operation. The order of operation is very important when simplifying algebraic expressions and equations. The result from problem two indicated that 6 (12.5 %) students were able to accurately understand the order of operation rules and wrote the correct answer to the question. The remaining 42 (87.5 %) students could not simplify the problem using a proper algorithm. This indicates that most of the students failed to know the rules of the order of operation.

Problem 3: 'Find the value of the given expression; $x^2 – 5x$, given the value of x = 5?'

The aim of this problem was to check the students' understanding of the role of mathematical symbols (coefficients, constant terms and exponents). In the process of simplifying algebraic expression or solving linear equation, the students were required to apply a succession of transformation rules in their multiplication of symbols (e.g., 2n), operation signs (e.g., +, -, x & ÷), exponents (e.g., x2) and variables which may appear as constants, exponents and coefficients or other rules in the equation. So, looking at the result from problem three only 2 (4 %) out of 48 (100%) students were able to solve the problem correctly. And the other 46 (96%) students did not solve the problem correctly. They rather attempted to solve like x2 – 5x = (5 x 2) – (5 x 5) = 10 – 25 = -15. So, this shows that most of the students did not know the rules of exponents and coefficients.

Problem 4: 'Fill-in certain numbers to equal the two expressions; such as: ☐ + 5 = ☐ + 2.

The aim of this problem was to check whether or not the students understand the equal relation. In this problem, an attempt was made to ask students to find a number that satisfies an equivalent equation. Moreover, this problem allowed more than one solution. Based on this, the researchers examined the students' conception of the meaning of equality. So, 31 (64.5 %) students were not aware of the meaning of equal relation as they could not write the correct answer. The other 17 (35.5 %) students used certain notations to represent the unknown number.

Problem 5: 'In the equation; x + 4 = 12, find the value of x?'

The aim of this problem was to examine the students' understanding of the inverse operation. In order to solve the problem, the students should remove 4 from both sides of the equation, this means creating zero pairs on the constant term of the given equation. However, the result of problem five indicated that only 37 (77 %) students answered the problem using a proper algorithm by selecting 8. But, out of 37 students, 29 students answered this problem using their memorization skill or reality. This means that the students worksheet shows that 8 + 4 = 12, so the value of x = 8, whereas the other 8 students solved this question by using inverse operation rule; x + 4 = 12; x + 4 – 4 = 12 – 4 = x = 8. The remaining 11 (23 %) students did not solve the problem correctly. Based on the students answer on the

worksheet, therefore, most of the students did not master the undoing process and creating zero pairs or emphasized memorization skills without its meaning.

Problem 6: 'The cost of one cloth is 93 birr; this cost is three times the cost of another cloth. Find the cost of the second cloth?'

The problem was designed to test the students' mathematical representation skill and conceptual understanding. The result of problem 6 indicated that 24 (50 %) students failed to understand the concept of the problem and they solved as the cost of the second cloth was; 93 x 3 = 279 birr. The remaining 24 (50 %) students understood the concept of the problem and wrote the correct answer. But out of 24 students, 11 students did not use certain notions to represent the unknown number; they rather used directly guess-and-check strategies.

Students' Pre-Intervention Interview

In addition to classroom observation and students pre-test, the researchers gathered data through student interview. The essence of this pre-intervention interview was to get the opinion of the students about the current teacher-centered methods of teaching mathematics in Ewket Fana Primary School and the students view about the mathematics subject. The researchers used the following pseudo names to represent the five students who participated in the interview. Student 1: Meseret, student 2: Adane, student 3: Almaze, student 4: Yared and student 5: Abebech. When asked student's opinion about mathematics as a subject, the students tried to disclose the following:

For me mathematics subject is very difficult as compared to other subjects. I do not like mathematics as it made me to memorize many formulas to solve mathematics questions (Meseret, pre-intervention interview data taken on February 8/ 2011).

In my view, mathematics subject is very difficult as it requires me to remember many formulas to solve mathematics problems. So, I personally feel that mathematics is a very difficult subject (Adane, pre-intervention interview data on February 8/2011).

I do not like mathematics subject as it is very difficult. It always requires me to do a lot of thinking and memorizing the formula or the procedures (Almaze, pre-intervention interview data on February 8/ 2011).

Mathematics is my favorite subject. This subject is simple to understand the concept and I like mathematics (Yared, pre-intervention interview data on February 8/2011).

I do not enjoy studying mathematics (Abebech, pre-intervention interview data on February 8/2011).

The researchers further asked their views about the connection between mathematics concepts learnt in school with the outside world. To this question, the students gave us the following response:

I do not think. Um… I think there is no connection between the two (Meseret, pre-intervention interview data on February 8/ 2011).

I do not know. Um….I think they have no connection (Adane, pre- intervention interview data on February 8/2011).

Somehow, yes. I only see some arithmetic like the one involving buying and selling (Almaze pre- intervention interview data on February 8/2011).

Yes, because all people use mathematics concepts by any means in their walk of life (Yared, pre-intervention interview data on February 8/2011).

No, it is hard to see. For me, I see buying and selling, but I really do not think that is mathematics (Abebech, pre-intervention interview data on February 8/2011).

The researchers asked the students about their attitude to learn mathematics in school. The students attest that:

Studying mathematics is very difficult; it's very hard for me to use many formulas to do mathematics problems (Meseret, pre-intervention interview data on February 8/2011).

I see mathematics subject as a hard subject. It is very difficult for me as compared to other subjects. (Adane, pre-intervention interview data on February 8/2011).

I think it is very difficult to learn mathematics as memorizing the formulas and procedures there in is very hard (Almaze, pre-intervention interview data on February 8/2011).

In the current situation, mathematics teaching and learning is well. I have a relatively better interest to mathematics subject as compared to other subjects (Yared, pre- intervention interview data on February 8/2011).

Studying mathematics subject is difficult as it requires remembering many formulas (Abebech, pre- intervention interview data on February 8/2011).

Table 2. Summary of The Pre-Intervention Interview

Questions	Number of students that gave a positive response	Number of students that gave a negative response
What is your opinion about mathematics as a subject?	1	4
Can you see any connection between the mathematics you learnt in school and in the real-world situation?	2	3
What is your opinion about the ways you are currently learning mathematics in school?	1	4

Table2 shows that most of the students did not have a positive impression of mathematics as a subject. Moreover, only one of the interviewed students had a positive attitude towards mathematics as a subject and the way they are currently learning mathematics in Ewket Fana Primary school. On the other hand, only two of the interviewed students were able to manage to draw some connection between the mathematics they are learning in school and their real-life situation. Generally, the preparation phase data showed that the students did not achieve the intention of the lesson and they failed to actively perform the learning activities. Moreover, these data emphasized that most of the students solved mathematical problems using guess-and-check strategies and they did not have a good attitude towards studying mathematics.

Enhancing Students' Understanding of Solving Linear Equation Using Balancing Model

In order to minimize the base line challenges, the researchers gave a series of activities related to the concept of solving linear equation with one variable to the students using the balancing model teaching method. This intervention involves 5 series of lessons with a pre- and post-tests. In this part, therefore, the researchers tried to reflect the development of the students' equation formulation skill, finding unknowns and how these two activities related to the balancing context during the learning process.

Lesson One: The Meaning of Equation And Equation Formulation

Before attending this lesson, the students learned about simplifying an algebraic expression as an introduction to linear equation. It is also important that the students knew about the meaning of variables, constant terms and mathematical operations. Although the students might have not used variables and constants, they found it in the form of balance or label in an algebraic expression. Therefore, the purposes of the present lesson were to sense equal sign and equal relations, formulate the equivalent equation using mathematical algorithms, and define the meaning of the equation. To achieve these objectives, two main activities were performed. They were defining equal relation and formulation of the equivalence equation. So, the researchers started the lesson by asking students thinking about the meaning of equality based on their real world experience. After the students were engaged with the context, the researchers asked them to seat in groups and measure the weight of 8 tiles on a real balance scale. This activity required them to enhance their concept of equal relation of two things they had performed and to make generalizations about the meaning of equality. All the groups put the same amount of tiles on both sides of the balancing scale. So, this activity simplified things to the students while measuring two equal things.

The second main activity was the formulation of the equivalent equation. In this part, however, the students were challenged to formulate equivalent equations through their own experiences. So, the researchers asked the students to explain and discuss the mathematical structure of the sum of seven and one unknown number is nine. At the onset, the students were confused to write this way in mathematical equation form. Some students answered that the unknown number is 2 because the sum of two and seven is nine. This indicates that the students understood facts or rules without understanding its concept. In order to enhance students understanding about the formulation of the equivalent equation, the researchers gave some additional instruction to the students using a balancing model to formulate the above problem in a mathematical equation form. To answer this question, the students put seven tiles and one cup on the right side and nine tiles on the left side of the balance scale. Using this context, most of the students came up with the idea of equation formulation. Thus are; $7 + x = 9$. After conducting this lesson, therefore, many students were able to formulate an equation using mathematical symbols.

Lesson Two: Finding The Unknowns

Based on what the students did in the first lesson, the researchers confirmed that the students were able to formulate an equation with mathematical structure. Their ability of formulating equations helped the students to understand the generalizations of numbers or representations of objects. During the first lesson, however, the students knew about only formulating a representation of mathematical equations. That is, they did not see built-in relationships of the concepts with quantities. In the second lesson, therefore, an attempt was made to realize the lesson objectives including using the rules of balancing scale to find-out the unknowns and the general formula of solving linear equation. To achieve these objectives, four sub-lessons were conducted including solving linear equation in the form of; $x + b = c$; $x – b = c$; $xb = c$ and solving multi-step linear equations when x is a variable and b and c are constant terms.

Solving Linear Equation in the Form of; x + b = c

Based on the analysis of the first lesson, the researchers concluded that most of the students achieved the objectives of the first lesson. Therefore, the researchers revised lesson one and moved to lesson two. In this lesson, two main activities were conducted; namely, finding the unknown number and generalizing the mathematical formula of solving for unknowns with addition. The researchers started the lesson by asking the students' prior understanding of equation formulation. The researchers asked the students to remove the weight on one side of the balance, and what they could do on the other side to keep the balance. Based on this question, the researchers expected the students to instill the idea of addition of an equation similar to this activity. The conjectures and expectations in this activity were that the students would remove the same number of tiles with various strategies. If they removed the same amount of tiles on both sides of the balance, the learning process would continue to the idea of addition of equation in balancing context.In this case, the students put tiles and cups on the balancing scale (tiles represent constant numbers and cups represent unknown numbers) and removed both sides of the balance scale the same amount of tiles. Through this attempt, they easily get the value of the unknown number. In this activity, therefore, most of the students removed the same number of tiles with different strategies and to find out the value of the variable by using a beam balance model.

After the students solved some contextual problems using a beam balance model, they moved to more mathematical situations, i.e., especially in finding the unknown in a given linear equation problem using possible strategies that were related to this context. Then, solving linear equation in the form of; $x + b = c$ as a learning activity was continued. So, the researchers asked the students to seat in a group and find the solution of the equivalent equation, $x + 3 = 9$. However, most of the students did not solve this problem using mathematical formulas. But, the students respond this activity saying that the value of $x = 6$, because $6 + 3 = 9$. It implies that the students emphasized memorization skill or realities without understanding its concept. Based on this, the researchers gave some additional learning experience to the students to solve this problem by using balancing models. As a result, most students express the idea in the following way.

Figure 5. Students' Argument and Answer For Linear Equation With Addition

As shown in Figure 5, most students have developed flexible strategies to solve linear equation problems after conducting solving addition of linear equation through balancing model.

Solving Linear Equation In The Form of x – b = c.

The context of this lesson was related to the previous lesson. So, the researchers started the lesson by reminding the concept of the previous lesson by simply asking the students understanding about the inverse operation. This idea is helpful to involve the students in the preparation of solving linear equation with subtraction. The students had informally experienced the inverse operation of subtraction in their walk of life. But, they did not connect this experience with the formal mathematical context. After the students' responses, the researchers provided additional problem to the students, which reads 'find the value of the unknown number in the equation; the difference between one unknown number and five is two'. To this problem, the students' response was that the value of the unknown number was seven because the difference between seven and five is two. This indicates that most students were mastering memorization skill without understanding its concept (see figure 4).

Figure 6. The Students' Gap In Solving Linear Equation With Addition And Subtraction

In order to enhance students' skill, the researchers gave some additional instruction to the students just to represent the above equivalent equation using different mathematical models and thereby to figure out the value of the unknown number. After the instruction, most students solved the problem as in the Figure 6.

Figure 7. The Students' Answer and Argument For Solving Linear Equation With Subtraction

In this activity, the students were intended to enhance their understanding of the concept of "zero pair" while solving any linear equations. For example, to solve an equation, x – 5 = 2, the students placed a yellow x and five negative unit pieces on the right side of the equation. In order to solve this equation, the students added five positive pieces for both sides of the equation. In so doing, the students created a zero pair on the left side of the equation which could be removed, leaving x = 8 as the solution to the equation.

Solving linear equation in the form of bx = c

The researchers began the lesson by reminding the topic of inverse operations again. The discussion, therefore, was focused on finding the inverse operations of multiplication and introducing solving linear equation with multiplication. In this case, the students listed the inverse operation of addition, subtraction, multiplication and division. Based on the students' response, the researchers gave additional activities for them, such as find the value of the unknown number in the equation, 2x = 6 and 5m = 2. These problems aimed to observe the students understanding of the inverse operation. After the students' discussion, some students solved those

problems based on informal strategies using guess-and-check strategies. It also assumed that the students did not know the undoing process of solving linear equation with multiplication. Afterwards, the researchers asked the students to represent and solve the first problem by using a beam balance model. During this occasion, the students were simply representing and solving the problem in the following ways.

Figure 8. Students' argument and answer for solving unknowns with multiplication

As it is indicated in figure 8, the researchers observed that the students developed different strategies to solve linear equations of any sort. When students learned how to solve multiplication equations, however, some students struggled to create the general formula of solving linear equation and some others emphasized memorization skill about the idea of "undoing" process. So, to enhance the students understanding of solving unknowns with multiplication, one of the researchers repeatedly taught the lesson by using balance model after solving multiplication of linear equation in the context of balancing scale. In the end, everyone became more experienced of the idea of undoing process and showed positive attitude towards the teaching-learning process.

Discussion of the Findings

Enhancing the students' conceptual understanding of linear equation

The students' pre-and post-test result confirmed that the performance of the students before the teaching experiment was not encouraging. That is, the students' pre-test indicated that most of the students did not solve the given problems by using a proper algorism. In contrast, the students' post-test result indicated that the students' performance become well as compared to the pre-test results. Furthermore, the students' post test result revealed that most of them were solving the post-test items using more flexible strategies as well as achieved a good result. In addition, the results in table 3 (M = 2.1 & SD = 1.6) revealed that the students' level of understanding was improved at a statistically significant level and the teaching intervention improved the students' result in linear equation.

On the other hand, classroom observation data revealed that most of the students used different strategies to solve equivalent equation problems and were becoming very reasonable for their work during the teaching experiment time. Moreover, most students had actively participated in the classroom activities and showed interest in the lesson study intervention. This situation shows that most of the students developed more flexible strategies to solve linear equation problems and more interested to the lesson study after the teaching experiment. Therefore, the implementation of the balancing model teaching method was more effective in enhancing the students' conceptual understanding of linear equations. This finding agrees with the findings of Caglayan and Olive (2010) who conducted a qualitative study among grade eight students (n = 24) solving of linear equation through balancing model using cups and tiles to represent variables and constants, respectively. So, Caglayan and Olive (2010) concluded that models and representations have developed the students' symbolic understanding of solving linear equation. In addition, this result agrees with the finding of Polly (2011) who examined grade three students understanding of solving linear equation using a beam balance model. The author concluded that the students have developed a better conceptual understanding of the lesson and developed their creative skill when using the balancing model of solving linear equation.

The other result obtained from observation data showed that the students who were taught through balancing model teaching method were becoming more and more confident from time to time. This finding is supported by earlier researchers (e.g., Warren & Cooper, 2005; Brizuela & Schliemann, 2004; Johnson, 1993). They described that the principles of the balance model and students seesaw game activities made them to see a connection between what is taught in school and students everyday life activities. Furthermore, the students' attempt of solving linear equations through the balancing context during the teaching experiment time made the students to engage into the lesson, allowed them to reflect on their learning and they were made to correct their errors in their thinking. Brizuela and Schliemann (2004) noted that the teaching methods employed by the model ease understanding of the concept and avoid mistakes when solving linear equations with one variable.

Generally, the results of this study showed that most students scored higher in the post-test. Moreover, most of the students came to actively participate in the classroom activities and developed their skills of solving linear equation through balancing model. This indicates that balancing model teaching approach was an effective tool to develop students' conceptual understanding and it also created a good learning situation. This finding agrees with the findings of early researchers (e.g., Caglayan & Olive, 2010; Schliemann, 2004; Polly, 2011; Vlassis, 2002; Johnson, 1993; Warren & Cooper, 2005). Those findings emphasized the effectiveness of the balancing model teaching approach in enhancing the concept of solving linear equation problems.

The state of sustainability of using balancing model teaching approach in Ewuket Fana Primary School

The observation data shows that many students developed positive attitude towards the lesson and they were happy with the learning process of solving linear equation through balancing model teaching method. This result corroborates the claim by Hopkins (2012) who suggests that education for sustainable development requires participatory teaching and learning methods that motivate and empower learners to change their behavior and take action for sustainable development. Generally, it was apparent at the beginning of any equation concept during this lesson that the students' attitudes about mathematics were improved.

The interview data also showed that the students changed almost entirely from the pre-intervention interview to the post-intervention interviews. During the pre-intervention interview, four students gave a negative response to the question 'why they were studying mathematics in school?', and after conducting the experiment through balancing model, all the students gave a positive response on the learning process of mathematics as a subject. On the other hand, the interview data from the teacher showed that he had developed a positive attitude towards the balancing model teaching method. The teacher further promised that he will use the balancing model teaching method in his future mathematics teaching. This result is in line with the findings of Johnson (1993). He described that using the balancing model to solve linear equations makes the classroom an interesting and exciting place for both teachers and learners as well.

4. CONCLUSION

Results revealed a significant improvement in students' performance after using the balancing model in teaching the concept of linear equation. Moreover, many students actively participated in the classroom activities and develop more flexible strategies when solving linear equation problems through balancing model. So, balancing model teaching method was considered important as a tool to developing students' conceptual understanding and performance of linear equations. The findings further showed that there was a possibility where alternative teaching strategies brought positive attitude towards the method of solving linear equation concepts among the students and the mathematics teacher.

Recommendations

Based on the findings of this study and the conclusions made thereof, the researchers forwarded the following recommendations:

- education officials should work on teachers capacity building and avail facilities for primary schools,
- curriculum designers should re-design mathematics textbooks in such a way that the textbooks will be smoothly integrated with the students' real-world situation,
- primary school mathematics teachers should apply balancing model teaching method in teaching abstract concepts like linear equation,
- mathematics teachers should assist their students' improvement of their attitude towards mathematics and give awareness to the students on how to relate mathematics concepts learned in class with real life, and
- future research should be done in the area of the current research emphasis at a large population and over a long period of time as well as other grade levels as well.

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